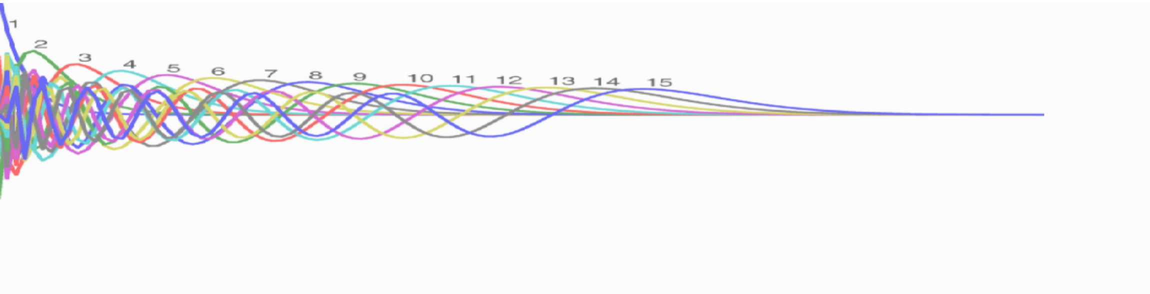
Calculation of Relative Weights in ADL Model

Internal Validation of Models
Santander Bank, Brazil



Distributed Lag Model

$$Y_t = \sum_{j\geq 0}^{\infty} v(j) X_{t-j} + \xi_{t-j} \, ,$$ ← **General Model**

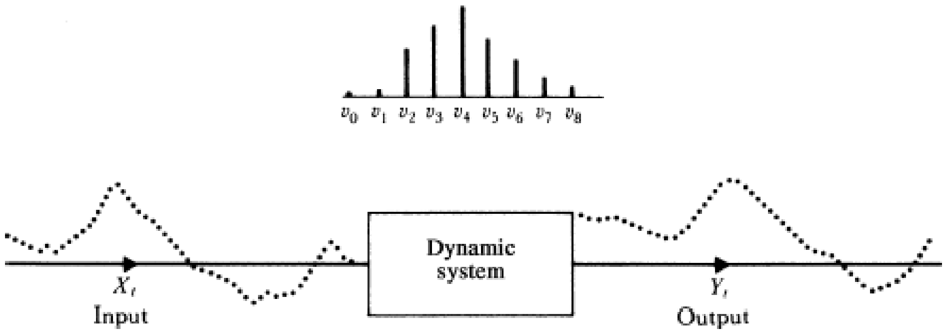
- (Y_t, X_t) is a dynamic system with adaptability and reaction (endogenous)
- $\xi_t \sim GP(0, \sigma^2)$ - Gaussian white noise process
- $v(j)$ is the jth impact coefficient
- $\sum_{j\geq 0}^{\infty} |v(j)| < \infty$ - Condition for the system to be dynamically stable (equilibrium)
- The existence of a long-run relationship between X_t and Y_t is a consequence of the ergodicity of the system
- The weights $v(j),\ j \geq 0$ are called **dynamic multiplier short-run** of shocks
- the accumulated weights $\sum_{j\geq 0}^{\infty} v(j)$ are called **dynamic multiplier log-run** of shocks

Distributed Lag Model

$$Y_t = v(0)X_t + v(1)X_{t-1} + v(2)X_{t-2} + \dots$$

$$= [v(0) + v(1)L + v(2)L^2 + \dots]X_t$$

- The polynomial $v(0) + v(1)L + v(2)L^2 + \dots$ is called **Transfer Function**
- The dynamics system (Y_t, X_t) is stable, if and only if, $v(0) + v(1)L + v(2)L^2 + \dots$ is convergente.



Simplified Case - The Koyck Distributed Lag (ADL Model)

- Assumption about the propagation → $v(j) = \beta\phi^j, \quad \beta \in \mathrm{R}, \quad |\phi| < 1.$

$$Y_t = \beta \sum_{j \geq 0}^{\infty} \phi^j X_{t-j} + \sum_{j \geq 0}^{\infty} \phi^j \xi_{t-j}\,,$$

Note that $|\phi| < 1$ is a necessary and sufficient condition for the system (Y_t, X_t) to be dynamically stable.

From the above equation, two equivalent representations emerge:

Representation Short-Run

$$Y_t = \beta \sum_{j\geq 0}^{\infty} \phi^j X_{t-j} + \sum_{j\geq 0}^{\infty} \phi^j \xi_{t-j} = \beta \sum_{j\geq 0}^{\infty} (\phi L)^j X_t + \sum_{j\geq 0}^{\infty} (\phi L)^j \xi_t ,= \quad \sum_{j\geq 0}^{\infty} (\phi L)^j \, (\beta X_t + \xi_t)$$

Note that $\sum_{j\geq 0}^{\infty} (\phi L)^j$ is a transfer function of system (X_t, Y_t). When $|\phi| < 1$ the system is stable

for which we have that $\sum_{j\geq 0}^{\infty} (\phi L)^j = \frac{1}{1-\phi L}$. Therefore, replacing in the equation presented above

we have that

$$Y_t = \frac{1}{1-\phi L} (\beta X_t + \xi_t) \;\rightarrow\; Y_t - \phi Y_{t-1} = \beta X_t + \xi_t$$

Conveniently organizing the terms we obtain the short-run representation:

$$Y_t = \phi Y_{t-1} + \beta X_t + \xi_t.$$

Based on the construction process, we know that the component ϕY_{t-1} condenses the past shocks from X to Y (before t). Therefore, the weight of that endogenous component has to be distributed so that we can calculate the relative importance of the exogenous variables.

Representation Log-Run

A simple way to divide the past effects of the endogenous component is by obtaining the long-term relationship (stationary relationship).

Of the short-sun representation we have that

$$Y_t = \frac{1}{1-\phi L}(\beta X_t + \xi_t) \rightarrow \; Y_t = \left(\frac{\beta}{1-\phi}\right) X_t + \xi_t^*$$

Conveniently organizing the terms we obtain the log-run representation:

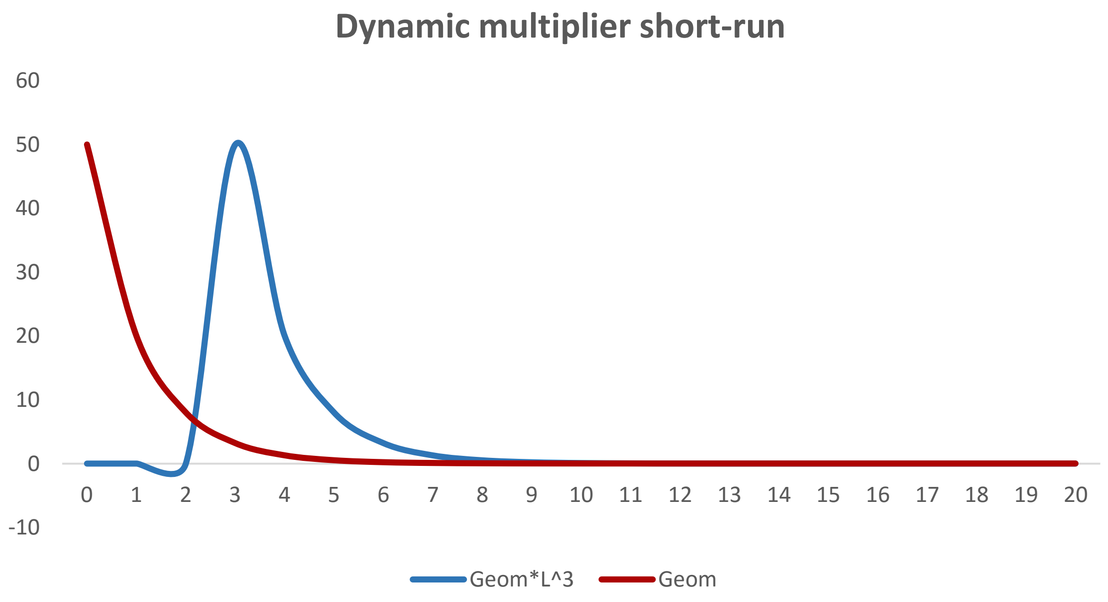
$$Y_t = \beta^* X_t + \xi_t^*.$$

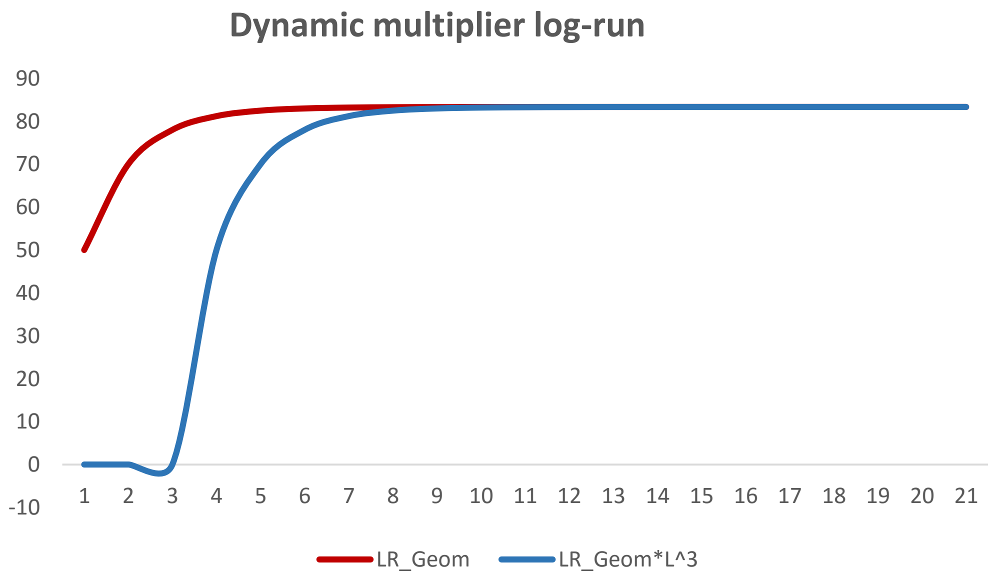
Where $\beta^* = \frac{\beta}{1-\phi}$ which is obtained by estimating ϕ and β through the short term representation. Note that this same procedure is easily generalizable for cases of more than one exogenous covariate.

It is important to clarify that the relative importance does not have to be influenced by the scales of the time series used, therefore, the estimation of ϕ and β must be obtained with all the series standardized previously and consequently without including the intercept.

Simplified Case - Shocks propagation

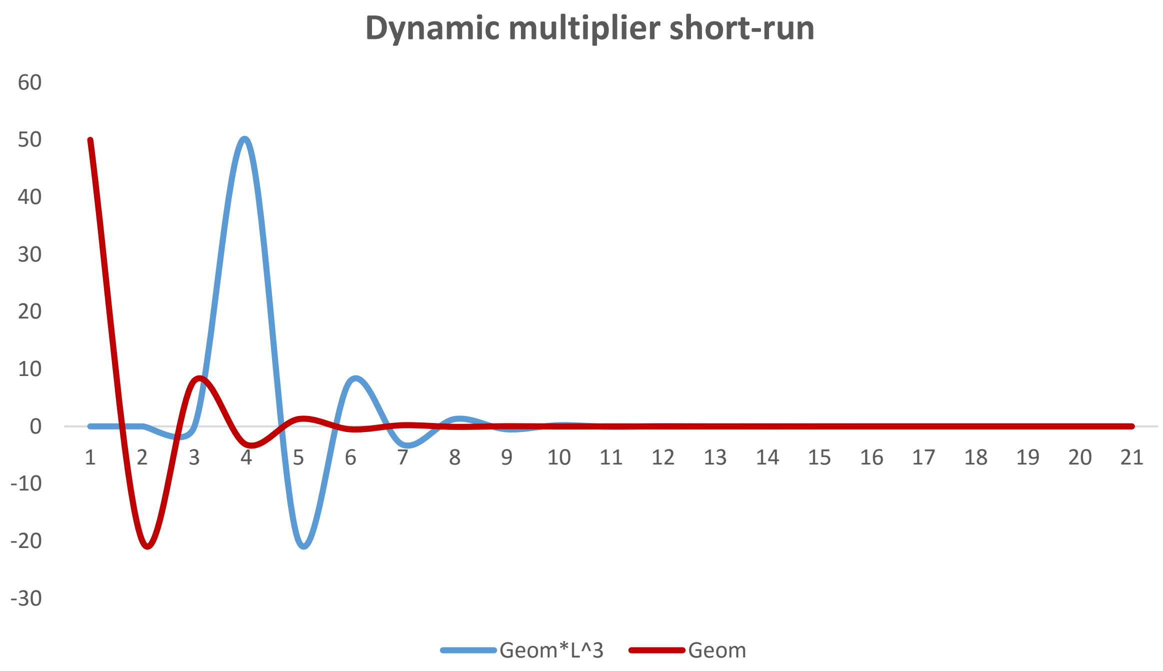
- If $\phi > 0$ → Fluactuaciones in the process of reversion to equilibrium.

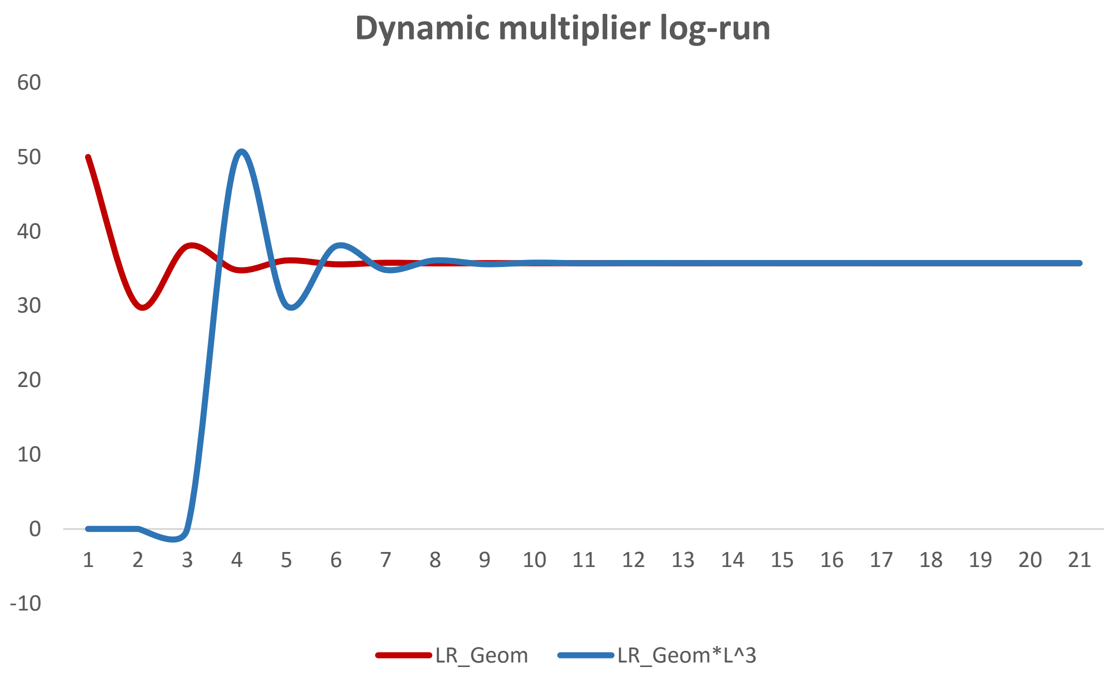




Simplified model - Fluctuations

- If $\phi < 0$ → Fluactuaciones in the process of reversion to equilibrium.





- In general, $|\phi|$ can be interpreted as a measure of system **resilience**.

Example: Fit of LGD - Portfolio X

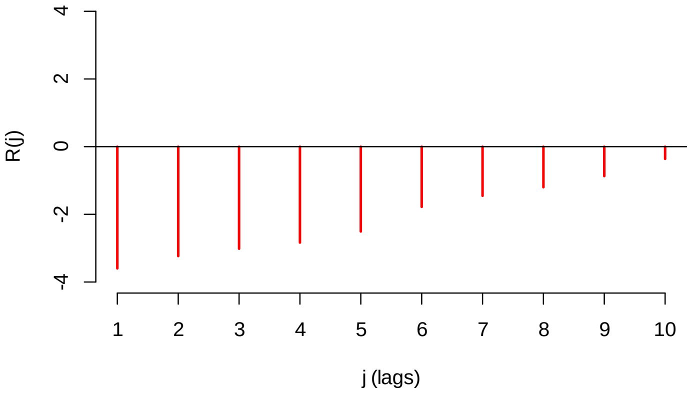
- Response variable ⟶ LOGIT_LDG
- The model used was an ADL equation with geometric propagation mechanism (simplified model)

Variáveis	*Beta*	*Erro Padrão*	*t Value*	*p-value*		Durbin-Watson		
					R2	*DW*	*dL*	*dU*
Intercepto	2,7802	0,78557	3,54000	1,80E-03	0,9452	2,1330	1,181	1,650
Logit_LGD_L1	0,3962	0,11818	3,35000	2,80E-03				
CDI_tri_tx	0,24946	0,04680	5,33000	<.0001				
PIB_lvl	-0,01583	0,00398	-3,98000	6,00E-04				

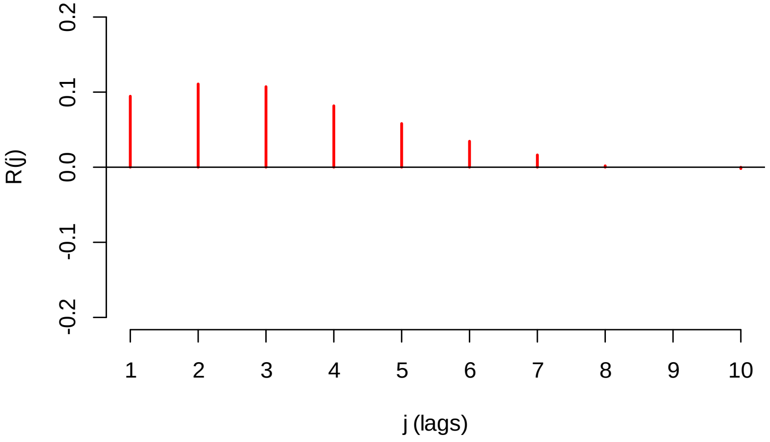
Estimativas dos parâmetros de Longo Prazo

Parâmetro	*Valor*
	0,41311584
	-0,02621512
	4,604073859
	0,014876622

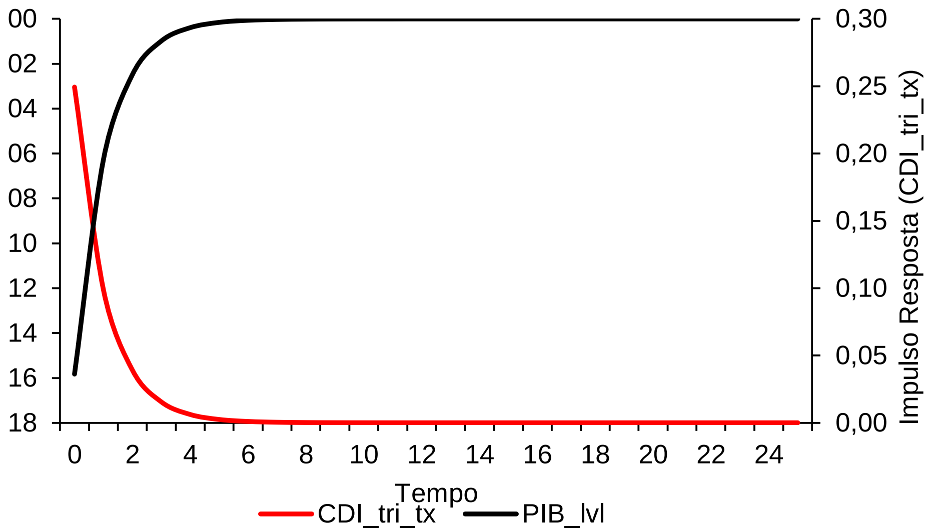
PBI ⟶ LGD



CDI ⟶ LGD

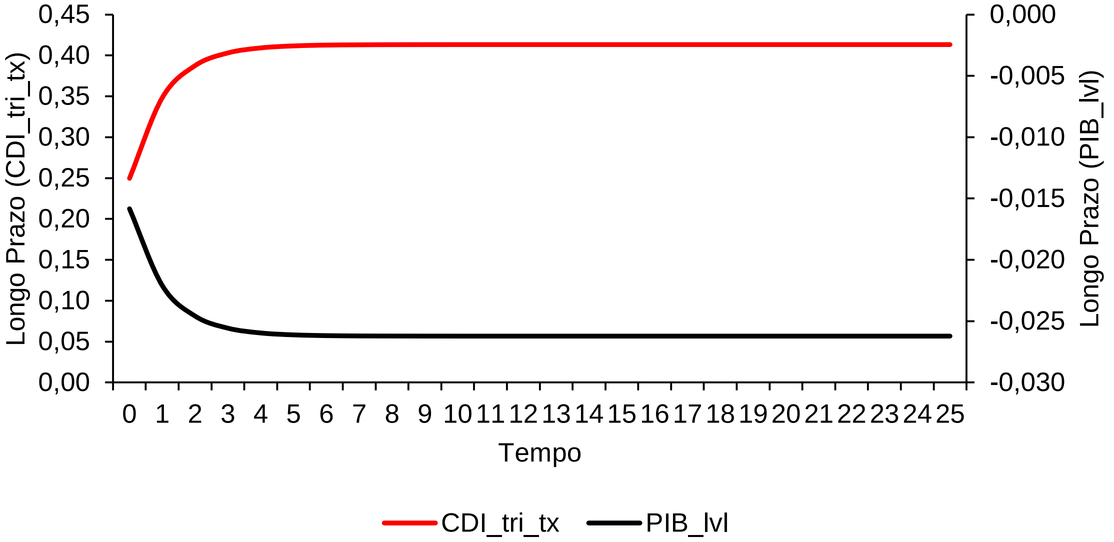


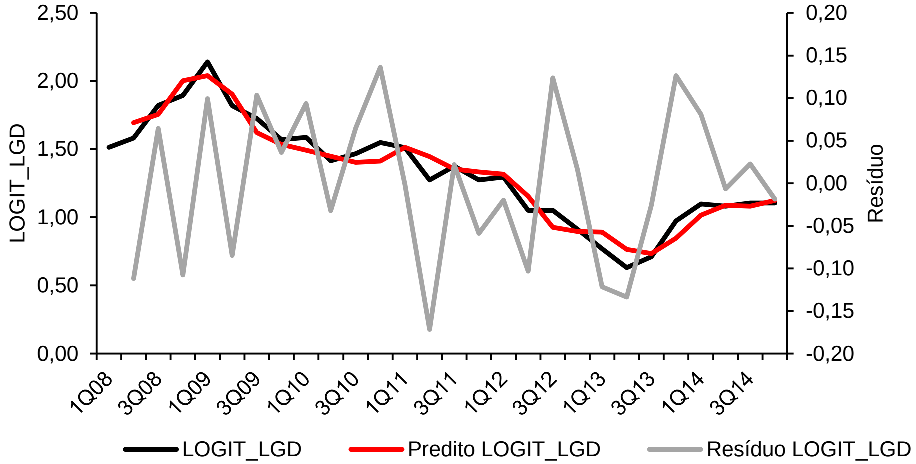
Short-run Dynamic Multiplier

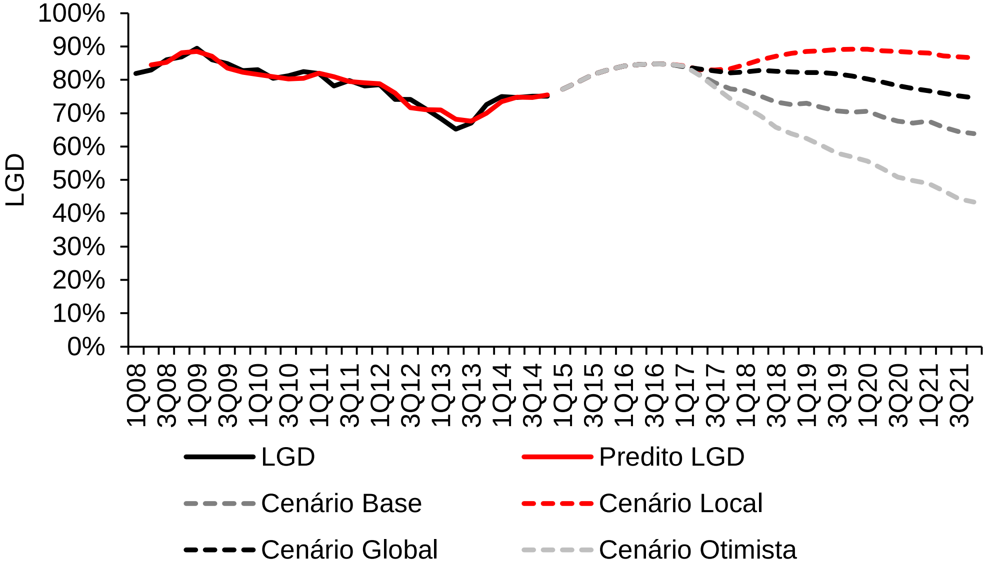


Example: Fit of LGD - Portfolio X

Log-run Dynamic Multiplier







- The long-term dynamic multiplier shows a stationary relationship between the variables which is condensed in the long-term parameters of the model.
- The residuals are stationary and have no temporal correlation.

Exemple: Calculation of Relative Importance

Fit with standardized series

Variable	Estimate	Std. Error	t value	Pr(>|t|)
pLOGIT_LGD_L1	0,39262	0,11628	3,37637	0,00250
pCDI_TRI	0,29304	0,05408	5,41906	0,00001
pPIB	-0,46125	0,11323	-4,07359	0,00044

Calculation of relative importance expressed in percentages

Variable	Estimate	abs Estimate	Percent
pCDI_TRI	0,4825	0,4825	38,85%
pPIB	-0,7594	0,7594	61,15%

References

1. **Transmission of Macroeconomic Shocks to Risk Parameters: Their uses in Stress Testing.** Helder Rojas and David Dias (2018). https://arxiv.org/abs/1809.07401

2. **Stress Testing Netwoks Reconstruction via Graphical Causal Model.** Helder Rojas and David Dias (2019).